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RADIATION OF AN ELEMENTARY SLOTTED DIPOLE LOCATED IN THE CENTER--ETC(U)
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FOREIGN TECHNOLOGY DIVISION

RADIATION OF AN ELEMENTARY SLOTTED DIPOLE LOCATED IN THE CENTER OF AN IDEALLY CONDUCTING DISK

By

Yu. V. Pimenov, L. G. Braude

EDITED TRANSLATION

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

RADIATION OF AN ELEMENTARY SLOTTED DIPOLE LOCATED IN THE CENTER OF AN IDEALLY CONDUCTING DISK

Obtained on the basis of the solution to the strict integral equation are the asymptotic expressions for the field appearing in the long-range zone with excitation of an ideally conducting disk by an elementary slotted dipole (magnetic dipole) located in the center of the disk. In the solution it was assumed that the radius of disk is much larger than the wavelength.

Introduction

The radiation of an elementary slotted dipole located in the center of an infinitely thin ideally conducting round disk was examined by M.G. Belkina in work [1]. The solution was obtained on the basis of Fourier method in the form of series according to spheroidal functions. As is known, such series in the case of the large (in comparison with the wavelength) disk merge extremely slowly, and the solution becomes practically unsuitable for the numerical calculations. Therefore, it is of interest to obtain the asymptotic solution for the case $\kappa a \gg 1$, where $\kappa = \frac{2\pi}{\lambda}$ is the wave number; λ - the wavelength; a - the radius of the disk.

A one-sided slot, cut in the disk, is equivalent to the elementary magnetic dipole lying on the disk. In virtue of the principle of duality [2], instead of solving the problem on the excitation of the disk by the elementary magnetic dipole located

in the center of the disk, it is possible to solve the problem on the excitation of an ideally conducting plane with a round opening by an electrical dipole located in the center of the opening and then, according to the known equations of transition, find the solution of the initial problem. When $\varkappa a \gg 1$ the second (additional) problem is solved considerably simply.

Statement of the problem

Let us examine the additional problem on the excitation of an ideally conducting plane with a round hole of radius a by an elemenatary electrical dipole with moment $\vec{p}$, located in the center of the hole.

Let us introduce the Cartesian coordinate system x, y and z, the origin of which coincides with the center of the hole, axis z is perpendicular to the plane of the screen, and the direction of the axis x coincides with the direction of the moment of the dipole $(\vec{p} = \vec{x}_0 p)$. Let us also introduce the cylindrical coordinate system r, φ, z, the axis z of which coincides with axis z of the Cartesian coordinate system (Fig. 1).

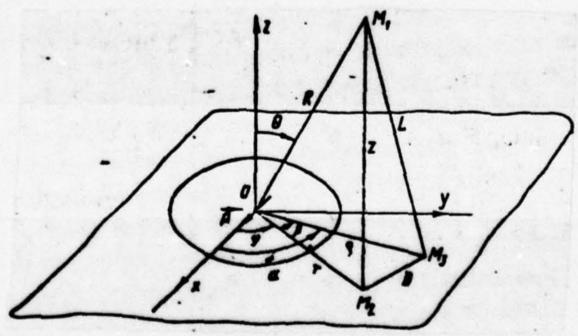

Fig. 1

The strength of the primarry electrical field created by the elementary electrical dipole

$$\vec{E}_1 = \vec{r}_0 E_{1r} + \vec{\varphi}_0 E_{1\varphi} + \vec{z}_0 E_{1z}, \qquad (1)$$

where

$$E_{1r}^{\circ} = f_1(r, z)\cos\varphi;\ E_{1\varphi}^{\circ} = f_2(r, z)\sin\varphi;\ E_{1z} = f_3(r, z)\cos\varphi;$$

$$f_1(r, z) = M\frac{e^{-i\kappa R}}{R^3}\left[z^2 - \frac{i}{\kappa R}\left(1 - \frac{i}{\kappa R}\right)(z^2 - 2r^2)\right];$$

$$f_2(r, z) = -M\frac{e^{-i\kappa R}}{R}\left[1 - \frac{i}{\kappa R} - \frac{1}{(\kappa R)^2}\right];$$

$$f_3(r, z) = -M\frac{e^{-i\kappa R}}{R}\frac{r}{R}\left[1 - \frac{3i}{\kappa R} - \frac{3}{(\kappa R)^2}\right]\frac{z}{R};$$

$$M = \frac{p\kappa^2}{4\pi\varepsilon};\ R = \sqrt{r^2 + z^2},$$

and ε is the dielectric constant of the medium. The dependence on time is taken in the form of $e^{i\omega t}$.

Under the effect of the field (1) on the plane with a round hole, there are applied the currents with density

$$\vec{j}(r, \varphi) = \vec{r}_0 j_r(r, \varphi) + \vec{\varphi}_0 j_\varphi(r, \varphi) = \vec{x}_0 j_x(r, \varphi) + \vec{y}_0 j_y(r, \varphi). \qquad (2)$$

The vector potential corresponding to these currents

$$\vec{A} = \frac{\mu}{4\pi}\int_a^\infty \rho\, d\rho \int_\varphi^{2\pi+\varphi} \frac{e^{-i\kappa L}}{L} j(\rho, \alpha)\, d\alpha, \qquad (3)$$

where

$$L = \sqrt{r^2 + \rho^2 + z^2 - 2r\rho\cos(\alpha - \varphi)},$$

and μ is the magnetic permeability of the medium.

G.A. Greenberg showed (see [3] or [4]) that in the case of ideally conducting infinitely thin screens, the vector potential $\vec{A}$ at points of the screen can be found irrespective of the function $\vec{j}(r, \varphi)$. This makes it possible, by applying the relation (3) to the points of the screen, to reduce the problem to the solution of the integral equation of the first kind. To determine the functions $\vec{A}$ on the screen, i.e., when $r > a,\ z = 0,$ let us proceed in the following manner.

The strength of the secondary electrical field $\vec{E}_1$ is connected with the vector potential $\vec{A}$ by the relation

$$\vec{E}_1 = -\operatorname{grad}\Psi - i\omega\vec{A}, \qquad (4)$$

where

$$\Psi = \frac{1}{\omega\varepsilon\mu}\operatorname{div}\vec{A}.$$

On the surface of the screen the following boundary conditions must be fulfilled:

$$E_{1r} = -E^{*}_{1r} \text{ when } r \geqslant a,\ z = 0; \tag{5}$$

$$E_{1\varphi} = -E^{*}_{1\varphi} \text{ when } r \geqslant a,\ z = 0, \tag{6}$$

which, by taking (4) into account, can be rewritten in the form:

$$\frac{\partial \Psi}{\partial r} + i\omega A_r = E^{*}_{1r} \text{ when } r \geqslant a,\ z = 0; \tag{7}$$

$$\frac{1}{r}\frac{\partial \Psi}{\partial \varphi} + i\omega A_\varphi = E^{*}_{1\varphi} \text{ when } r \geqslant a,\ z = 0, \tag{8}$$

where A_r and A_φ are the radial and azimuthal components of the vector $\vec{A}$, respectively.

Since $E^{*}_{1r} = f_1(r, z)\cos\varphi$, and $E^{*}_{1\varphi} = f_2(r, z)\sin\varphi$, then, according to results of work [3], the scalar potential Ψ on the surface of the screen can be presented in the form

$$\Psi = \psi(r)\cos\varphi \text{ when } r \geqslant a,\ z = 0, \tag{9}$$

where the function $\psi(r)$ must satisfy the condition of the radiation and the differential equation

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{d\psi}{dr} + \left(\kappa^2 - \frac{1}{r^2}\right)\psi = -\frac{\partial f_2}{dz}\Big|_{z=0};\ r \geqslant a. \tag{10}$$

Solving (10) by the method of the variation of the arbitrary constants and considering the condition of radiation, we get

$$\psi(r) = BH_1^{(2)}(\kappa r) + \frac{\pi i}{4}\left\{H_1^{(1)}(\kappa r)\int_r^\infty F(t)H_1^{(2)}(\kappa t)\,dt - \right.$$

$$\left. - H_1^{(2)}(\kappa r)\int_r^\infty F(t)H_1^{(1)}(\kappa t)\,dt\right\},\ r \geqslant a, \tag{11}$$

where

$$F(t) = -M\frac{e^{-i\kappa t}}{t^2}\left[1 - \frac{3i}{\kappa t} - \frac{3}{(\kappa t)^2}\right],$$

$H_1^{(1)}$ and $H_1^{(2)}$ are Hankel's functions of the first order of the first and second kind, respectively, and B is a certain constant which must be defined later from the condition of vanishing on the edge of the hole of the radial component of the current density:

$$j_r(a) = 0. \tag{12}$$

Thus the function $\psi(r)$, and, consequently, function $\Psi(r, z)$ when $r \geqslant a;\ z=0$ are determined with an accuracy up to the constant B.

Expressing from (7) and (8) components A_r and A_φ and then going over to the Cartesian component of vector $\vec{A}$, we get

$$\vec{A} = \vec{x}_0 A_x + \vec{y}_0 A_y, \tag{13}$$

where

$$\left.\begin{aligned} A_x &= A_x^{(0)}(r) + A_x^{(2)}(r)\cos 2\varphi \text{ при } r \geqslant a;\ z=0 \\ A_y &= A_y^{(2)}(r)\sin 2\varphi \text{ при } r \geqslant a;\ z=0 \end{aligned}\right\}; \tag{14}$$

$$\left.\begin{aligned} A_x^{(0)}(r) &= \frac{1}{2\omega}\left[\frac{d\psi}{dr} - f_1(r,\ 0) + \frac{1}{r}\psi + f_2(r,\ 0)\right] \\ A_x^{(2)}(r) = A_y^{(2)}(r) &= \frac{1}{2\omega}\left[\frac{d\psi}{dr} - f_1(r,\ 0) - \frac{1}{r}\psi - f_2(r,\ 0)\right] \end{aligned}\right\}. \tag{15}$$

Applying (3) to points of the screen $(r \geqslant a,\ z=0)$ and considering (13), we arrive at two independent integral equations of the first kind:

$$A_x^{(0)}(r,\ 0) + A_x^{(2)}(r,\ 0)\cos 2\varphi = \frac{\mu}{4\pi}\int_a^\infty \rho\, d\rho \int_\varphi^{\varphi+2\pi} \frac{e^{-ikD}}{D} j_x(\rho,\ \alpha)\, d\alpha; \tag{16}$$

$$A_y^{(2)}(r,\ 0)\sin 2\varphi = \frac{\mu}{4\pi}\int_a^\infty \rho\, d\rho \int_\varphi^{\varphi+2\pi} \frac{e^{-ikD}}{D} j_y(\rho,\ \alpha)\, d\alpha, \tag{17}$$

where

$$D = \sqrt{r^2 + \rho^2 - 2r\rho\cos(\alpha - \varphi)}.$$

From the form of the left sides of equations (16) and (17), it follows that the functions $j_x(\rho,\ \alpha)$ and $j_y(\rho,\ \alpha)$ can be sought in the form

$$\left.\begin{aligned} j_x &= j_x^{(0)}(\rho) + j_x^{(2)}(\rho)\cos 2\alpha \\ j_y &= j_y^{(2)}(\rho)\sin 2\alpha \end{aligned}\right\}, \tag{18}$$

where $j_y^{(2)}(\rho = j_x^2\, \rho)$.

Substituting (18) into (16) and going in the internal integral to the new variable of integration β according to the equation $\beta = \alpha - \varphi$, we arrive at the two independent integral equations of the first kind for functions $j_x^{(0)}(\rho)$ and $j_x^{(2)}(\rho)$:

$$A_x^{(\nu)}(r,\ 0) = \frac{\mu}{4\pi}\int_a^\infty j_x^{(\nu)}(\rho)\,\rho\, d\rho \int_0^{2\pi} \frac{e^{-ikD}}{D}\cos\nu\beta\, d\beta;\ r \geqslant 0;\ \nu = 0;\ 2. \tag{19}$$

where

$$D=\sqrt{r^2+\rho^2-2r\rho\cos\beta}.$$

The left sides of equations (19) are known with an accuracy to the constant B, which must be defined after the finding of functions $j_x^{(0)}(\rho)$ and $j_x^{(2)}(\rho)$. The conditon (12) serves for the calculation of constant B, and after the transition to functions $j_x^{(0)}(\rho)$ and $j_x^{(2)}(\rho)$ takes the form

$$j_x^{(0)}(a)+j_x^{(2)}(a)=0. \tag{20}$$

Thus the problem on the excitation of an ideally conducting plane with a round hole by an elementary electrical dipole located in the center of the hole is reduced to the solution of two independent integral equations of the first kind (19) with an additional condition (20).

Determination of Currents

Equations (19) are strict integral equations of the problem. They are correct at any values of the parameter κa. The solution to these equations when $\kappa a \gg 1$ is of interest to us. In this case the left sides of equations (19), defined by equations (15), are considerably simplified. Since $\kappa a \gg 1$, and $r \geqslant a$, then the Hankel functions, which enter into the left sides of equations (19), can be replaced by the first terms of their asymptotic expansions:

$$H_\nu^{(2)}(\kappa r)\approx\frac{\sqrt{2}}{\sqrt{\pi\kappa r}}e^{-i\kappa r}e^{i\left(\frac{\pi}{4}-\frac{\nu\pi}{2}\right)}. \tag{21}$$

Disregarding, furthermore, terms of the order of $\frac{1}{\kappa r}$ in comparison with unity, we get

$$\begin{aligned}&\omega p\left[B'\frac{e^{-i\kappa r}}{\sqrt{r}}-\delta_\nu i\frac{e^{-i\kappa r}}{\sqrt{r}}\right]=\\&=\int_a^\infty j_x^{(\nu)}(\rho)\rho\,d\rho\int_0^{2\pi}\frac{e^{-i\kappa D}}{D}\cos\nu\beta\,d\beta;\ r>a;\ \nu=0;\ 2,\end{aligned} \tag{22}$$

where

$$B'=B\frac{2\sqrt{2\pi}\,e^{i\frac{3\pi}{4}}}{p\kappa\sqrt{\kappa}}\,\varepsilon;\quad \delta_\nu=\begin{cases}1 & \text{when } \nu=0,\\0 & \text{when } \nu=2.\end{cases}$$

The internal integral on the right side of (22) can be transformed by using the asymptotic equality proven in work [5]:

$$\int_0^{2\pi} \frac{e^{-i\kappa D}}{D} \cos\nu\beta\, d\beta = -\frac{\pi i}{\sqrt{r\rho}} \{H_0^{(2)}(\kappa|r-\rho|) + i(-1)^\nu H_0^{(2)}[\kappa(r+\rho)]\} + O[(\kappa a)^{-3/2}], \quad \nu = 0;\ 2. \tag{23}$$

Substituting (23) into (22) and introducing the dimensionless variables ξ, η and γ connected with ρ, r and k by relations

$$\rho = a(1+\xi), \quad r = a(1+\eta), \quad \gamma = \kappa a, \tag{24}$$

we obtain

$$\int_0^\infty u^{(\nu)}(\xi) H_0^{(2)}(\gamma|\eta-\xi|)\, d\xi + i\int_0^\infty u^{(\nu)}(\xi) H_0^{(2)}[\gamma(\eta+\xi+2)]\, d\xi = C e^{i\frac{\pi}{4}} \frac{\sqrt{2}}{\sqrt{\pi\gamma}} e^{-i\gamma\eta} - \delta_\nu H_0^{(2)}[\gamma(\eta+1)], \tag{25}$$

where

$$u^{(\nu)}(\xi) = \frac{\sqrt{2\pi}\, e^{-i\frac{\pi}{4}} a^2}{i\omega\sqrt{\gamma\rho}} j_z^{(\nu)}[a(1+\xi)]\sqrt{1+\xi}, \quad \nu = 0;\ 2, \tag{26}$$

and $C = -i\sqrt{a}\, e^{-i\gamma} B'$ is a certain constant which will be defined subsequently from condition (20).

Using the equalities proven by G.A. Greenberg [6]

$$\int_0^\infty \frac{e^{-i\gamma(\xi+R_1)}\sqrt{R_1}}{\pi\sqrt{\xi}(\xi+R_1)} H_0^{(2)}(\gamma|\eta-\xi|)\, d\xi = H_0^{(2)}[\gamma(\eta+R_1)], \tag{27}$$

$$\int_0^\infty \frac{\sqrt{\gamma}\, e^{-i\frac{\pi}{4}}}{\sqrt{2\pi\xi}} e^{-i\gamma\xi} H_0^{(2)}(\gamma|\eta-\xi|)\, d\xi = e^{-i\gamma\eta}. \tag{28}$$

we transform the equations (25) into the integral equations of the second kind:

$$u^{(\nu)}(\xi) = -i\frac{e^{-i\gamma(\xi+2)}}{\pi\sqrt{\xi}} \int_0^\infty \frac{u^{(\nu)}(t)\, e^{-i\gamma t}\sqrt{t+2}}{\xi+t+2}\, dt + C\frac{e^{-i\gamma\xi}}{\pi\sqrt{\xi}} - \delta_\nu \frac{e^{-i\gamma(\xi+1)}}{\pi\sqrt{\xi}(\xi+1)}, \quad \nu = 0;\ 2. \tag{29}$$

Since according to the assumption that $\gamma = \kappa a \gg 1$, the solution to equations (29) can be found by the method of

successive approximations. However, it is more convenient to use the artificial method.

Functions $u^{(\nu)}(\xi)$ are proportional to the component $j^{(\nu)}[a(1+\xi)]$ of the density of the current induced on the screen. With an increase in the variable ξ, functions $u^{(\nu)}(\xi)$ decrease according to the absolute value and vanish when $\xi\to\infty$. Therefore when $\gamma\gg 1$ the main contribution to the value of the integral entering into (29) is given by the neighborhood of the point $\xi = 0$. Consequently, there takes place the following approximate equality:

$$u^{(\nu)}(\xi) = -i\frac{\sqrt{2}}{\pi}\frac{e^{-i\gamma(\xi+2)}}{\sqrt{\xi}(\xi+2)}U_0^{(\nu)} + C\frac{e^{-i\gamma\xi}}{\pi\sqrt{\xi}} - \\ -\delta_\nu\frac{e^{-i\gamma(\xi+1)}}{\pi(\xi+1)}, \quad \nu = 0;\ 2, \tag{30}$$

where

$$U_0^{(\nu)} = \int_0^\infty u^{(\nu)}(\xi)e^{-i\gamma\xi}d\xi, \quad \nu = 0;\ 2. \tag{31}$$

We can strictly show that the error of equality (30) does not exceed $O(\gamma^{-3/2})$.

For determining the constants $U_0^{(\nu)}$, let us multiply both sides of (30) by $e^{-i\gamma\xi}$ and integrate with respect to ξ from zero to infinity. As a result let us impart to two (for $\nu = 0$ and $\nu = 2$) independent algebraic equations, the solving of which we get

$$U_0^{(\nu)} = \frac{\dfrac{Ce^{i\frac{\pi}{4}}}{\sqrt{2}\sqrt{\pi\gamma}} - \delta_\nu[1-\Phi(\sqrt{i2\gamma})]}{1+ie^{i2\gamma}[1-\Phi(\sqrt{i4\gamma})]}, \quad \nu = 0;\ 2, \tag{32}$$

where

$$\Phi(\sqrt{iw}) = \frac{2e^{i\frac{\pi}{4}}}{\sqrt{\pi}}\int_0^{\sqrt{w}} e^{-is^2}ds. \tag{33}$$

It remained to determine the constant C. Using condition (20), which after the transition to functions $u^{(\nu)}(\xi)$ takes the form

$$u^{(0)}(0) + u^{(2)}(0) = 0, \tag{34}$$

we obtain

$$C=\frac{e^{-i\gamma}}{2}\,\frac{1+i\,e^{i2\gamma}\left[1-\Phi\left(\sqrt{i4\gamma}\right)\right]-\frac{i}{\sqrt{2}}\left[1-\Phi\left(\sqrt{i2\gamma}\right)\right]}{1+i\,e^{i2\gamma}\left[1-\Phi\left(\sqrt{i4\gamma}\right)\right]-i\,\frac{e^{-i2\gamma}\,e^{-i\pi/4}}{2\sqrt{\pi\gamma}}}. \qquad (35)$$

Expression (35) is considerably simplified if function $\Phi\left(\sqrt{iw}\right)$ is replaced by its asymptotic representation. Here there will be fulfillled the simple relation $C=\frac{1}{2}e^{-i\gamma}+O\left(\gamma^{-3/2}\right)$.

Thus the functions $u^{(\nu)}(\xi)$ are completely defined, and, consequently, the distribution of the currents induced on the screen is known.

Determination of the Field

Let us turn to the determination of the field which appears with the excitation of an ideally conducting plane with a round hole by an elementary electrical dipole located in the center of the hole.

The vector potential of the currents induced on the screen is expressed by the equation (3) and has two components: A_x and A_y, where

$$\left.\begin{aligned} A_x &= A_x^{(0)}(r,\ z)+A_x^{(2)}(r,\ z)\cos 2\varphi \\ A_y &= A_y^{(2)}(r,\ z)\sin 2\varphi;\ A_y^{(2)}=A_x^{(2)} \end{aligned}\right\}. \qquad (36)$$

On the screen $(r\geqslant a,\ z=0)$ functions $A_x^{(\nu)}(r,\ z)$ coincide with the functions $A_x^{(\nu)}(r)$, introduced earlier and at the arbitrary point in space are determined by the expression

$$A_x^{(\nu)}=-\frac{\mu}{4\pi}\int_a^\infty j_x^{(\nu)}(\rho)\,\rho\,d\rho\int_0^{2\pi}\frac{e^{-i\kappa L}}{L}\cos\nu\beta\,d\beta,\quad \nu=0;\ 2 \qquad (37)$$

Equation (37) is inconvenient for the numerical calculations. Let us find the asymptotic representation. Here we will distinguish two regions: first, the one adjoining the z axis and the second, the remaining part of the space.

First, let us examine the second region. Let us introduce the spherical coordinate system R, θ, ϕ, the polar axis of which coincides with the z axis of the cylindrical coordinate system (Fig. 1).

Equation (37) in this coordinate system takes the form

$$A_z^{(\nu)} = \frac{i e^{i\frac{\pi}{4}} \omega \mu p \sqrt{\gamma}}{4\pi \sqrt{2\pi a}} \int_0^{\infty} u^{(\nu)}(\xi) \sqrt{1+\xi}\, d\xi \int_0^{2\pi} \frac{e^{-i\gamma_0 L_0}}{L_0} \times$$
$$\times \cos \nu\beta\, d\beta, \quad \nu = 0;\ 2, \tag{38}$$

where

$$L_0 = L/a = [\, r_0^2 + (1+\xi)^2 - 2r_0 (1+\xi) \sin\theta \cos\beta]^{1/2},$$
$$r_0 = R/a.$$

Since in the examined region the inequality $\gamma \sin\theta \gg 1$, is fulfilled, then the internal integral in (38) can be transformed according to the equation (see work [7])

$$\int_0^{2\pi} \frac{e^{-i\gamma L_0}}{L_0} \cos \nu\beta\, d\beta = -\frac{\pi i}{\sqrt{r_0 (1+\xi) \sin\theta}} \{H_0^{(2)}(\gamma b) +$$
$$+ i(-1)^{\nu} H_0^{(2)}(\gamma d)\} + O[(\gamma \sin\theta)^{-3/2}], \quad \nu = 0;\ 2, \tag{39}$$

where

$$b = [\, r_0^2 + (1+\xi)^2 - 2r_0 (1+\xi) \sin\theta]^{1/2},$$
$$d = [\, r_0^2 + (1+\xi)^2 + 2r_0 (1+\xi) \sin\theta]^{1/2}.$$

Substituting into (38) values of functions $u^{(\nu)}(\xi)$ from (30) and applying to equation (39), we get

$$A_z^{(\nu)} = -\frac{i e^{i\frac{\pi}{4}} \omega \mu p \sqrt{\gamma}}{4 \sqrt{2\pi \sin\theta}\, a \sqrt{r_0}} \{U_0^{(\nu)} [Q_1(2,\ r_0,\ \theta) +$$
$$+ iQ(2,\ r_0,\ \theta+\pi)] + iC [Q_2(r_0,\ \theta) + iQ_2(r_0,\ \theta+\pi)] -$$
$$- i\delta_{\nu} [Q_1(1,\ r_0 \theta) + iQ_1(1,\ r_0,\ \theta+\pi]\}, \tag{40}$$

where

$$Q_1(\sigma,\ r_0,\ \theta) = \frac{1}{\pi} \int_0^{\infty} \frac{e^{-i\gamma(\xi+\sigma)} \sqrt{\sigma}}{\sqrt{\xi}\,(\xi+\sigma)} H_0^{(2)}(\gamma b)\, d\xi; \tag{41}$$

$$Q_2(\sigma, r_0,\ \theta) = \frac{1}{\pi} \int_0^{\infty} \frac{e^{-i\gamma\xi}}{\sqrt{\xi}} H_0^{(2)}(\gamma b)\, d\xi. \tag{42}$$

The integral $Q_1(\sigma,\ r_0,\ \theta)$ was examined in detail in work [7]. In the long-range zone (when $r_0 \to \infty$) the following asymptotic equality is correct:

$$Q_1(\sigma, r_0, \theta) = \frac{\sqrt{2}\, e^{-i\frac{\pi}{4}}}{\sqrt{\pi\gamma}} \frac{e^{-i\gamma r_0}}{\sqrt{r_0}} e^{-i\gamma(\sigma-1)\sin\theta} \{1 - \Phi(\sqrt{i\gamma\sigma(1-\sin\theta)})\} + O(r_0^{-3/2}). \quad (43)$$

The integral $Q_2(r_0, \theta)$ is calculated in work [8] and equal to

$$Q_2(r_0, \theta) = \frac{\sqrt{2}}{\sqrt{\pi\gamma}} e^{i\frac{\pi}{4}} e^{-i\gamma(r_0-1)\sin\theta} [1 - \Phi(\sqrt{i\gamma r_0(1-\sin\theta)})]. \quad (44)$$

In the long-range zone (when $r_0 \to \infty$) expression (44) takes the form

$$Q_2(r_0, \theta) = \frac{\sqrt{2}}{\pi\gamma} \frac{e^{-i\gamma r_0}}{\sqrt{r_0}} \frac{e^{i\gamma\sin\theta}}{\sqrt{1-\sin\theta}} + O(r_0^{-3/2}). \quad (45)$$

Using the relations (43) and (45) and going from components $A_x^{(0)}$ and $A_x^{(2)}$ in the Cartesian coordinate system to components A_φ and A_θ in the spherical coordinate system, we get

$$A_\varphi = \frac{\omega\mu\, p}{4\pi a} \frac{e^{-i\gamma r_0}}{r_0} S_\varphi(\theta) \sin\varphi; \quad (46)$$

$$A_\theta = \frac{\omega\mu\, p}{4\pi a} \frac{e^{-i\gamma r_0}}{r_0} S_\theta(\theta) \cos\varphi, \quad (47)$$

where

$$S_\varphi(\theta) = \frac{1}{\sqrt{\sin\theta}} \{[U_0^{(2)} - U_0^{(0)}] F_1(\theta) + F_2(\theta)\}; \quad (48)$$

$$S_\theta(\theta) = \frac{\cos\theta}{\sqrt{\sin\theta}} \Big\{ [U_0^{(2)} + U_0^{(0)}] F_1(\theta) - F_2(\theta) + C \frac{2e^{-i\frac{\pi}{4}}}{\sqrt{\pi\gamma}} \Big(i \frac{e^{i\gamma\sin\theta}}{\sqrt{1-\sin\theta}} - \frac{e^{-i\gamma\sin\theta}}{\sqrt{1+\sin\theta}} \Big) \Big\}; \quad (49)$$

$$F_1(\theta) = e^{-i\gamma\sin\theta} [1 - \Phi(\sqrt{i 2\gamma(1-\sin\theta)})] + i e^{i\gamma\sin\theta} [1 - \Phi(\sqrt{i 2\gamma(1+\sin\theta)})];$$

$$F_2(\theta) = i[1 - \Phi(\sqrt{i\gamma(1-\sin\theta)})] - [1 - \Phi(\sqrt{i\gamma(1+\sin\theta)})].$$

The strength of the secondary electrical field $\vec{E}_1$ in the long-range zone is connected with the vector potential $\vec{A}$ by the relation $\vec{E}_1 = -i\omega\vec{A}$. Consequently, the components of the vector of the strength of the total electrical field $\vec{E} = \vec{E}_1 + \vec{E}_1^0$ in the long-range zone in region $\gamma\sin\theta \gg 1$ are equal, respectively, to

$$E_\varphi = -\frac{e^{-i\gamma r_0}}{r_0}\frac{i\gamma^2 p\sin\varphi}{4\pi a^2 \varepsilon}[S_\varphi(\theta) - i]; \quad (50)$$

$$E_\theta = -\frac{e^{-i\gamma r_0}}{r_0}\frac{i\gamma^2 p\cos\varphi}{4\pi a^2 \varepsilon}[S_\theta(\theta) + i]. \quad (51)$$

Let us turn to the computation of the field in the region adjoining the z axis, and let us be limited to the examination of the long-range zone.

Assuming in (38) that $L_0 \approx r_0 - (1+\xi)\sin\theta\cos\beta$ and changing the order of integration, we get

$$A_x^{(\nu)} = \frac{i e^{i\frac{\pi}{4}}\omega\mu p\sqrt{\gamma}}{4\pi\sqrt{2\pi}a}\frac{e^{-i\gamma r_0}}{r_0}\int_0^{2\pi} G^{(\nu)}(\beta)e^{i\gamma\sin\theta\cos\beta}\times$$

$$\times\cos\nu\beta\, d\beta, \quad \nu = 0;\ 2, \quad (52)$$

where

$$G^{(\nu)}(\beta) = \int_0^\infty u^{(\nu)}(\xi)\sqrt{1+\xi}\,e^{i\gamma\xi\sin\theta\cos\beta}d\xi, \quad \nu = 0;\ 2. \quad (53)$$

Integral (53) can be computed asymptotically. Substituting the values of functions $u^{(\nu)}(\xi)$ from (30) into (53) and disregarding terms of the order $O\{[\gamma(1-\sin\theta)]^{-3/2}\}$, we get

$$G^{(\nu)}(\beta) = \frac{1}{\sqrt{2\pi\gamma}}\frac{e^{-i\frac{\pi}{4}}}{\sqrt{1-\sin\theta\cos\beta}}K^{(\nu)} + O\{[\gamma(1-\sin\theta)]^{-3/2}\}, \quad (54)$$

where

$$\left.\begin{aligned} K^{(0)} &= -iU_0^{(0)}e^{-i2\gamma} + \sqrt{2}C - \sqrt{2}e^{-i\gamma} \\ K^{(2)} &= -iU_0^{(2)}e^{-i2\gamma} + \sqrt{2}C \end{aligned}\right\}. \quad (55)$$

Expanding $(1-\sin\theta\cos\beta)^{-1/2}$ in power series of $\sin\theta\cos\beta$ and being limited to the first three terms of the expansion, after the term-by-term integration in equation (52) let us impart to the following expression:

$$A_x^{(\nu)} = \frac{i\omega\mu p}{4\pi a}\frac{e^{-i\gamma r_0}}{r_0}K^{(\nu)}T^{(\nu)}(\theta), \quad \nu = 0;\ 2. \quad (56)$$

where

$$T^{(0)}(\theta) = J_0(\gamma\sin\theta) + \frac{1}{2}\sin\theta J_1(\gamma\sin\theta) +$$

$$+\frac{3}{16}\sin^2(\theta)[J_0(\gamma\sin\theta) - J_2(\gamma\sin\theta)]; \quad (57)$$

$$T^{(2)}(\theta) = -J_2(\gamma\sin\theta) + \frac{1}{4}\sin\theta\,[J_1(\gamma\sin\theta) -$$

$$-J_3(\gamma\sin\theta)] + \frac{3}{32}\sin^2\theta\,[J_0(\gamma\sin\theta) - 2J_2(\gamma\sin\theta) + J_4(\gamma\sin\theta)]. \quad (58)$$

Here J_n is the Bessel function of the first kind of order n. In going over to the components A_φ and A_θ in the spherical coordinate system, we get

$$A_\varphi = \frac{i\omega\mu p}{4\pi a}\,\frac{e^{-i\gamma r_0}}{r_0}\,V_\varphi(\theta)\sin\varphi; \quad (59)$$

$$A_\theta = \frac{i\omega\mu p}{4\pi a}\,\frac{e^{-i\gamma r_0}}{r_0}\,V_\theta(\theta)\cos\varphi, \quad (60)$$

where

$$V_\varphi(\theta) = K^{(2)}T^{(2)}(\theta) - K^{(0)}T^{(0)}(\theta); \quad (61)$$

$$V_\theta(\theta) = [K^{(2)}T^{(2)}(\theta) + K^{(0)}T^{(0)}(\theta)]\cos\theta. \quad (62)$$

Consequently, the components of the vector of the total electrical field strength in the long-range zone in region $\gamma(1-\sin\theta) \gg 1$ are equal to

$$E_\varphi = \frac{e^{-i\gamma r_0}}{r_0}\,\frac{\gamma^2 p\sin\varphi}{4\pi a^2\varepsilon}\,[V_\varphi(\theta) - 1]; \quad (63)$$

$$E_\theta = \frac{e^{-i\gamma r_0}}{r_0}\,\frac{\gamma^2 p\cos\varphi}{4\pi a^2\varepsilon}\,[V_\theta(\theta) + 1]. \quad (64)$$

Thus the additional problem on the excitation of an ideally conducting plane with a round hole by an elementary electrical dipole, located in the center of the hole, is completely solved. Let us turn to an analysis of the initial problem.

Excitation of the Disk by an Elementary Magnetic Dipole

The electromagnetic field created by the elementary magnetic dipole (one-way slot), located in the center of an ideally conducting disk, can be found by the duality principle [2], using the obtained solution. Here in the long-range zone in the region $\gamma\sin\theta \gg 1$ the total electrical field strength $\vec{E}'$ is determined by the following expressions:

a) in the upper half-space (z 0):

$$E'_\theta = H'_\varphi\sqrt{\frac{\mu}{\varepsilon}} = -\frac{e^{-i\gamma r_0}}{r_0}\,\frac{i\gamma^2 m\sin\varphi}{4\pi a^2\varepsilon}\,[S_\varphi(\theta) - 2i], \quad (65)$$

$$E'_\varphi = H'_\theta\sqrt{-\frac{\mu}{\varepsilon}} = -\frac{e^{-i\gamma r_0}}{r_0}\,\frac{i\gamma^2 m\sin\varphi}{4\pi a^2\varepsilon}\,[S_\varphi(\theta) + 2i]. \quad (66)$$

where m is the moment of the dipole;

b) in the lower half-space ($z < 0$):

$$E'_\theta = H'_\varphi \sqrt{\frac{\mu}{\varepsilon}} = \frac{e^{-i\gamma r_0}}{r_0} \frac{i\gamma^2 m \sin\varphi}{4\pi a^3 \varepsilon} S_\varphi(\theta), \qquad (67)$$

$$E'_\varphi = H'_\theta \sqrt{\frac{\mu}{\varepsilon}} = \frac{e^{-i\gamma r_0}}{r_0} \frac{i\gamma^2 m \cos\varphi}{4\pi a^3 \varepsilon} S_\theta(\theta). \qquad (68)$$

Correspondingly, in the region adjoining the z axis [i.e., with the fulfillment of equality $\gamma(1-\sin\theta) \gg 1$], the field in the long-range zone is determined by the equations:

a) in the upper half-space:

$$E'_\theta = H'_\varphi \sqrt{\frac{\mu}{\varepsilon}} = \frac{e^{-i\gamma r_0}}{r_0} \frac{\gamma^2 m \sin\varphi}{4\pi a^3 \varepsilon} [V_\varphi(\theta) - 2], \qquad (69)$$

$$E'_\varphi = H'_\theta \sqrt{\frac{\mu}{\varepsilon}} = \frac{e^{-i\gamma r_0}}{r_0} \frac{\gamma^2 m \cos\varphi}{4\pi a^3 \varepsilon} [V_\theta(\theta) + 2]; \qquad (70)$$

b) in the lower half-space:

$$E'_\theta = H'_\varphi \sqrt{\frac{\mu}{\varepsilon}} = -\frac{e^{-i\gamma r_0}}{r_0} \frac{\gamma^2 m \sin\varphi}{4\pi a^3 \varepsilon} V_\varphi(\theta), \qquad (71)$$

$$E'_\varphi = H'_\theta \sqrt{\frac{\mu}{\varepsilon}} = -\frac{e^{-i\gamma r_0}}{r_0} \frac{\gamma^2 m \cos\varphi}{4\pi a^3 \varepsilon} V_\theta(\theta). \qquad (72)$$

Numerical Results

For a comparison of the obtained asymptotic expressions (65)-(72) with results of the strict solution [1], the numerical calculations for the case $\gamma = 5$ were conducted.

Figure 2 gives the normalized radiation pattern of the elementary magnetic dipole located in the center of an ideally conducting disk on the upper side of the disk corresponding to the plane $\phi = 90°$. The solid line shows the values of E'_θ of the component referred to the maximal value of modulus $|E'_\theta|$, taken from work [1] (strict solution). Applied by a dashed line are similar values computed according to equations (65), (67), (69), and (71).

Figure 3 gives the normalized radiation pattern in the plane $\phi = 0°$. The solid line corresponds to the strict solution and the dashed line to values computed according to equations (66), (68), (70), and (72).

Figure 4 shows the normalized radiation pattern of the elementary magnetic dipole located in the center of an ideally conducting disk in the plane $\phi = 90°$ calculated according to equations (65), (67), (69), and (71) when $\gamma = 10$.

Figure gives the normalized radiation pattern in the plane $\phi = 0°$ calculated by equations (66), (68), (70), and (72) when $\gamma = 10$.

On Figs. 6 and 7 similar patterns are plotted when $\gamma = 15$.

As the calculations show, equations (65)-(72) overlap the whole range of the change in angle θ.

The obtained solution will be more accurate, the larger the quantity $\gamma = ka$. However, as the numerical calculations show, it satisfactorily transfers the character of the radiation pattern at such a comparatively small value of γ, as $\gamma = 5$.

The obtained solution is suitable only when the magnetic dipole lies on the disk; however, the method used in the work makes it possible to obtain the solution also for the case of the magnetic dipole raised slightly above the disk.

In conclusion the authors wish to thank Professor G.Z. Ayzenberg and Assistant Professor L.S. Korol'kevich for their discussion of this work and their valuable advice.

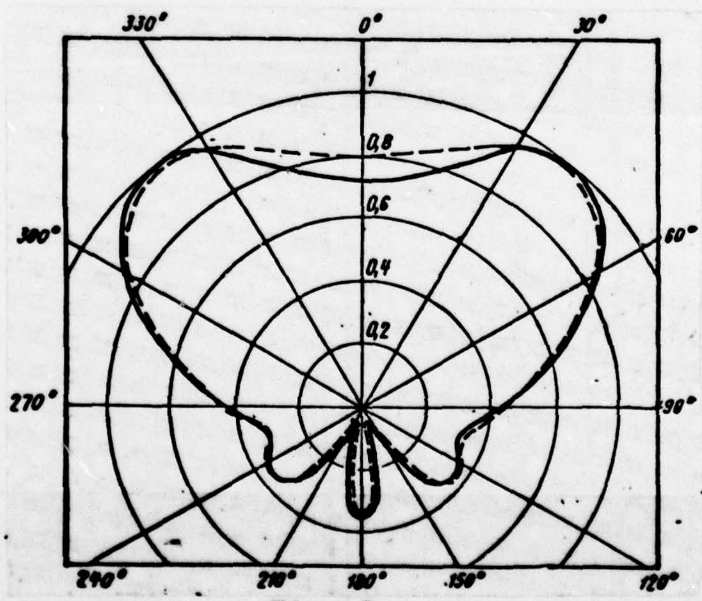

Fig. 2

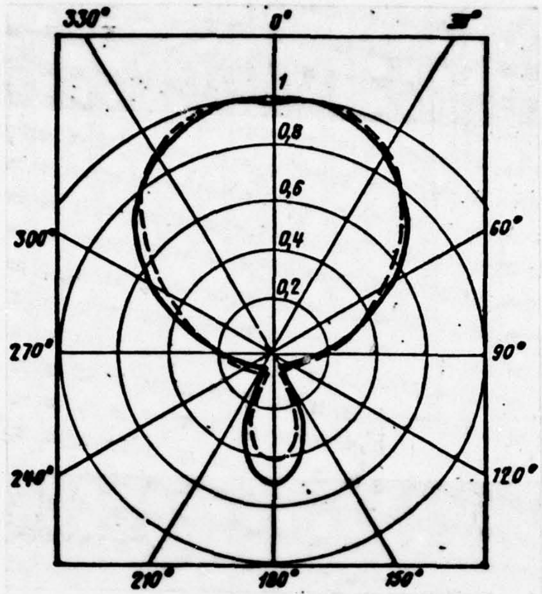
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Fig. 3

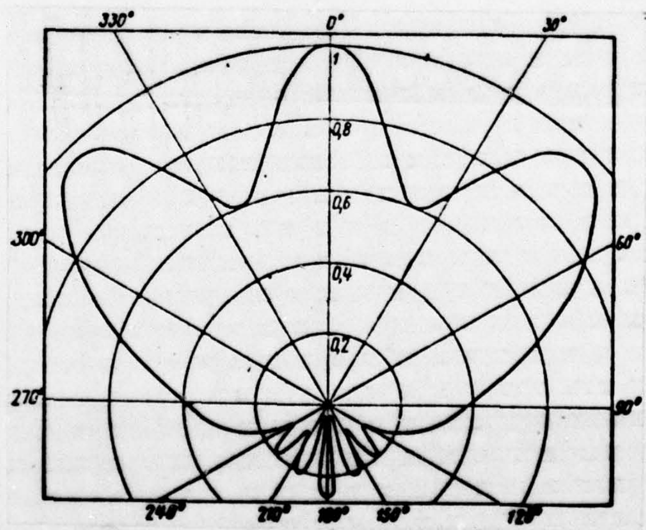
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Fig. 4

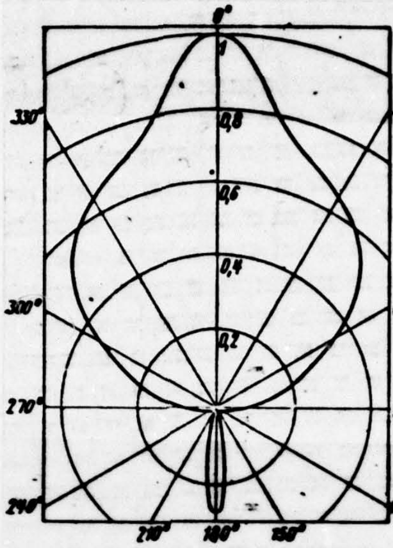
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Fig. 5

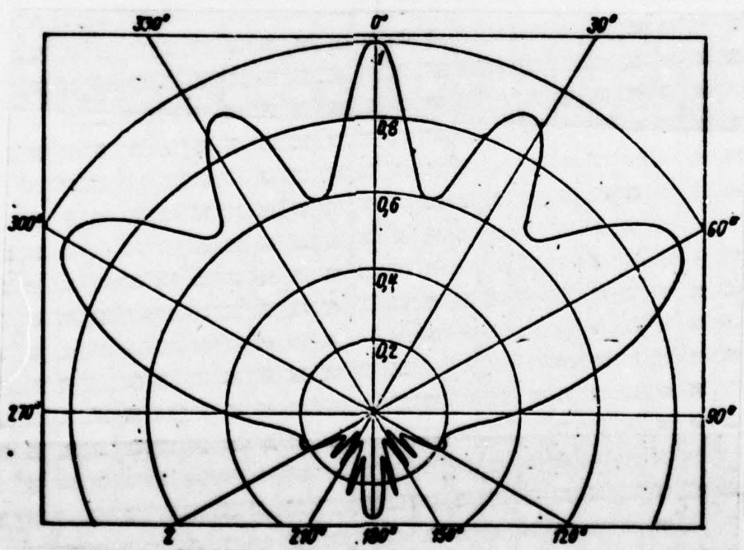
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Fig. 6

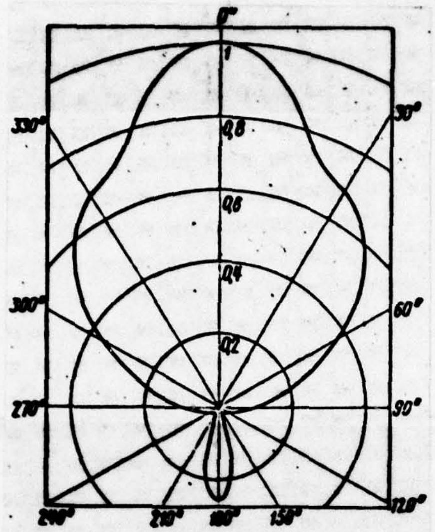

Fig. 7

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